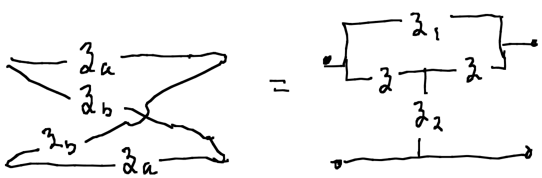
Note Title 9/19/2020

ENEE 610
11/05/20

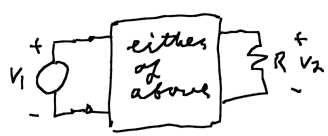
signups
corrections
exponential impulse response matrix

sensitivity
adjoint circuit



$$y_a = 2y_1 + y, \quad z_b = 2z_2 + z \qquad y_1 = \tfrac{1}{2}(y_a - y), \quad y = 1/z, \quad z_2 = \tfrac{1}{2}(z_b - z)$$

If $z = R = 1/G$
then the
bridge tee
is constant R



$$\Rightarrow \frac{V_2}{V_1} = \frac{1 - z_a/R}{1 + z_a/R} = \frac{1 - G/(2y_1+G)}{1 + G/(2y_1+G)} = \frac{y_1/G}{1 + y_1/G}$$

passive if z_1 is PR
all-pass if z_a is LPR

Ex: $\frac{V_2}{V_1} = \frac{s}{s+2} \Rightarrow y_1 = Gs, \; y_a = G(2s+1)$ is PR

Ex: $\frac{V_2}{V_1} = \frac{s^2 - 4s + 2}{s^2 + 4s + 2} \Rightarrow z_a = R\left(\frac{4s}{s^2+2}\right)$; $y_1 = G\left(\frac{s^2+2}{4s} - 1\right)$
is LPR
$= G\left(\frac{s^2 - 4s + 2}{4s}\right)$
not PR

Ex: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \delta \begin{bmatrix} x_{0_1} \\ x_{0_2} \end{bmatrix}$; $\mathcal{L}[x] = \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right)^{-1} x_0$

$$\begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} = \frac{1}{s(s+3)+2} \begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix} = \frac{1}{s^2+3s+2} \begin{bmatrix} s+3 & +1 \\ -2 & s \end{bmatrix}$$

$$= \frac{1}{s+1} K_1 + \frac{1}{s+2} K_2 \Rightarrow K_1 = \frac{s+1}{(s+1)(s+2)} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \Bigg|_{s=-1} = \frac{1}{1} \begin{bmatrix} -1+3 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$$

$$K_2 = \frac{1}{s+1} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \Bigg|_{s=-2} = -1 \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$= \frac{1}{s+1} \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} + \frac{1}{s+2} \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$$

$\therefore$ Using the causal response, $s = \sigma + j\omega$ for $\sigma > -1$

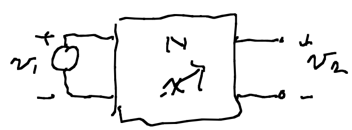
$$e^{\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}t} 1(t)x_0 = \left\{ \begin{bmatrix}2 & 1\\ -2 & -1\end{bmatrix} e^{-t} + \begin{bmatrix}-1 & -1\\ 2 & 1\end{bmatrix} e^{-2t} \right\} 1(t)\, x_0$$

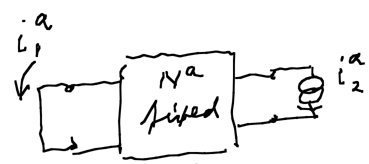
$$\dot{x} = Ax + Bu$$

Sensitivity $\quad T(s) = \frac{V_2}{V_1}$ or any transfer function

$$S_x^T = \frac{dT/dx}{T/x}$$

can use an adjoint circuit

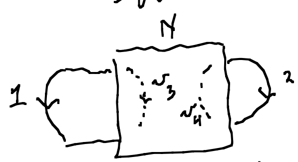




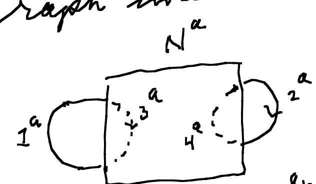
adjoint circuit

$$Y^a = Y^T$$

both have the graph in each $\quad v_b^T i_b = 0 = v_b^{aT} i_b^a = v_b^{aT} i_b = v_b^T i_b^a$



$$\begin{bmatrix} i_3 \\ i_4 \end{bmatrix} = Y \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}$$



$$\begin{bmatrix} i_3^a \\ i_4^a \end{bmatrix} = Y^a \begin{bmatrix} v_3^a \\ v_4^a \end{bmatrix}$$

$$v_b^a i_b - v_b^T i_b^a = \left(v_1^a i_1 + v_2^a i_2 + [v_3^a\ v_4^a]\, Y \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}\right) - \left(v_1 i_1^a + v_2 i_2^a + [v_3\ v_4]\, Y^a \begin{bmatrix} v_3^a \\ v_4^a \end{bmatrix}\right)$$

take $\dfrac{d}{dx} = \bullet$

$$\left(\dot v_1^a i_1 + v_1^a \dot i_1 + \dot v_2^a i_2 + v_2^a \dot i_2 + \dot v_p^{aT} Y v_p + v_p^{aT} \dot Y v_p + v_p^{aT} Y \dot v_p\right)$$

$$- \left(\dot v_1 i_1^a + v_1 \dot i_1^a + \dot v_2 i_2^a + v_2 \dot i_2^a + \dot v_p^T Y^a v_p^a + v_p^T \dot Y^a v_p^a + v_p^T Y^a \dot v_p^a\right) = 0$$

$v_p = \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}$

$$= \Big(\underset{\substack{\text{short} \\ @ v_1^a}}{0} + 0 + \underset{\substack{\text{(open} \\ @ v_2)}}{0} + 0 + \underset{\substack{\text{no } x \\ \text{variation} \\ \text{in } N^a}}{0} + v_p^{aT} \dot Y v_p + v_p^{aT} Y \dot v_p\Big)$$

$$- \Big(\underset{\substack{\text{input} \\ v_1 \text{ no} \\ \text{change} \\ \text{with } x}}{0} + \underset{\substack{N^a \\ \text{fixed}}}{0} + \underset{\substack{\uparrow \\ \text{apply} \\ \text{so} \neq 0}}{\dot v_2 i_2^a} + \underset{\substack{N^a \\ \text{fixed}}}{0} + \dot v_p^T Y_a v_p^a + \underset{\substack{N^a \\ \text{fixed}}}{0} + \underset{\substack{N^a \\ \text{fixed}}}{0}\Big) = 0$$

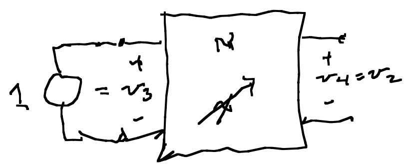
fix $v_1 = 1$ then

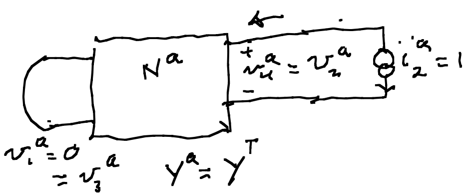
$$v_p^{aT} \dot Y v_p + \underbrace{v_p^{aT} Y \dot v_p - \dot v_p^T Y^a v_p^a} = \dot v_2 i_2^a$$

$$+ \underbrace{\dot v_p^T Y^T v_p^a - \dot v_p^T Y^a v_p^a}$$

$$\underbrace{\dot v_p^T \left[Y^T - Y^a \right] v_p^a}_{0} \quad \text{choose } Y^a = Y^T$$

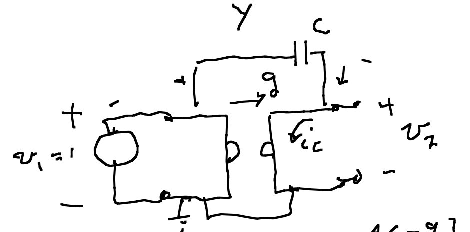
$$\frac{d\, v_2/v_1}{dx} = \frac{d v_2}{dx} \cdot \underset{\substack{\| \\ \text{choose } i_2^a = 1}}{\frac{1}{1}} = \underbrace{v_p^{aT} \frac{dY}{dx} \cdot v_p = \frac{d\left(v_2/v_1\right)}{dx}}$$



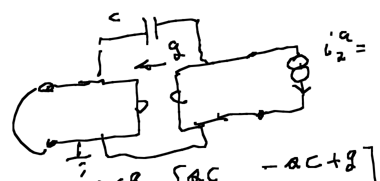


$Y^a = Y^T$

Ex:



$$Y = \begin{bmatrix} sC & -sC-g \\ -sC+g & sC \end{bmatrix}$$



$$Y^a = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$$

$\Longrightarrow$

$i_C^a = C s v_2^a = -i_2^a = -1$

$v_2^a = -1/Cs$

If desire $\dfrac{d\, v_2/v_1}{dg}$; $\dfrac{dY}{dg} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

desire $\begin{bmatrix} v_3 & v_4 \end{bmatrix}^a \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix}$ with $v_3^a = 0$, $v_4^a = v_2^a = -1/Cs$, $v_3 = v_1 = 1$, $v_4 = v_2$ desired for $\dfrac{v_2}{v_1}$

$$= \begin{bmatrix} 0 - \frac{1}{Cs} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ v_2 \end{bmatrix} = -1/Cs = \frac{d\, v_2/v_1}{dg}$$

$$S_g^{v_2/v_1} = \frac{(-1/Cs)}{v_2/v_1 / g}$$

$\therefore$ need $T(s) = v_2/v_1$

gyrator current $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_{\text{gyrator}} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{\text{gyrator}} \Longrightarrow i_2 = g \cdot v_1 = g =$ cap. current

$v_{cap} = \dfrac{1}{Cs} \cdot i_2 = g/Cs$

$$v_2 = v_{cap} + v_1 = q/C_a + 1 \Rightarrow \frac{v_2}{v_1} = 1 + q/C_a$$